Fission and cluster decay of ^{76}Sr nucleus in the groud-state and formed in heavy-ion reactions

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Abstract: Calculations for fission and cluster decay of ^{76}Sr are presented for this nucleus to be in its ground-state or formed as an excited compound system in heavy-ion reactions. The predicted mass distribution, for the dynamical collective mass transfer process assumed for fission of ^{76}Sr, is clearly asymmetric, favouring α-nuclei. Cluster decay is studied within a preformed cluster model, both for ground-state to ground-state decays and from excited compound system to the ground-state(s) or excited states(s) of the fragments.

$^{76}_{38}Sr$ is a superdeformed nucleus with an estimated quadrupole deformation $\beta_2 = 0.44$ (see Fig. 1 in [1]). From the point of view of known **spherical** shell closures at Z=N=40, such a large ground-state deformation for ^{76}Sr means the natural breaking of these spherical shells and hence nuclear instability against both fission and exotic cluster decay processes. However, we shall see in the following that, like the other superdeformed nuclei in this mass region [1], though this nucleus is naturally stable (negative Q-value) against only light clusters with masses $A_2 < 12$, the calculated cluster decay half lives for $A_2 \geq 12$ are also large enough $(T_{1/2} > 10^{80}s)$ to term this nucleus as a stable nucleus against all cluster decays. This kind of stability could apparently be due to stable **deformed** shell closures at Z=N=38, predicted earlier in many other calculations [2,3]. Alternatively, if these nuclei are prepared in heavy-ion collisions, then, depending on the excitation energy of the compound nucleus formed, both fission (also called, fusion-fission) and cluster decay are the viable processes. The present day experiments are directed at these studies (see e.g. [4-7] and earlier references there in).

It now seems accepted that compound systems with $A \leq 42$ are characterized by nuclear orbiting phenomena (the deep inelastic process), although a considerable amount of yield due to fusion-fission could not be ruled here too [8,9]. On the other hand, the systems with $A \approx 47 - 60$ are strongly the cases of fusion-fission process (fully energy-damped fragments) since for all the cases studied so-far the observed yields are independent of nuclei in the

entrance channel and no strong peaking of yields is observed near the target and projectile masses [4-7]. In all these cases, asymmetric mass splitting is favoured and hence lie far below the Businaro-Gallone transition point [10] (the fissility parameter $x(= Z^2/50A)$ is less than the $x_{BG} = 0.396$ for $\ell = 0$ and this value decreases as ℓ-value increases). For nuclei with $A \sim 80$, the situation is not so clear. Only three experiments are made [11,12] that form the compound systems $^{78}_{38}Sr$, $^{80}_{40}Zr$ and $^{83}_{36}Kr$ and one calculation is available for ^{80}Zr [13]. Notice that fissility x =0.370 and 0.312 ($< x_{BG}$), respectively, for ^{78}Sr and ^{83}Kr but x =0.40($> x_{BG}$) for ^{80}Zr. Also, at least ^{78}Sr and ^{80}Zr are superdeformed in their respective ground-states. The interesting result is: whereas the measured mass spectra of ^{83}Kr [11] and ^{80}Zr [12] are clearly asymmetric and symmetric, as expected, respectively, but that of ^{78}Sr [12] is more asymmetric than symmetric. The last experiment on $^{28}Si +^{50}Cr$ forming the compound system $^{78}Sr^*$ is made only at one energy ($E_{lab} = 150 MeV$) and at one angle ($\theta_{lab} = 30^0$). In view of this result we have chosen ^{76}Sr nucleus with x(=0.38) as well as deformation β larger than that for ^{78}Sr (estimated $\beta_2 = 0.41$ for ^{78}Sr). Such a study has not yet been taken up either experimentally or theoretically. Our calculations in the following show a clear asymmetric mass distribution, as expected from the point of view of Businaro-Gallone transition [10]. This result suggests that a properly angle intergrated mass distribution for ^{78}Sr (not yet measured) should also be clearly asymmetric since x for ^{78}Sr is smaller than that for ^{76}Sr.

Theoretically, both the quantum mechanical fragmentation process [13-16] as well as the statistical models [8,11,17-19] have been used to explain the measured mass distributions in these reactions. The statistical or compound nucleus model calculations, assuming fusion-fission process, are made for three possible cases of (i) two spheres separated by a fixed distance d=2fm [11], (ii) saddle point shapes, called transition state model [8], and (iii) scission point shapes with d taken as a variable [17-19]. In this last case, the Hauser-Feshbach formalism [20] is extended to include the decay fragments heavier than α-particle. Perhaps, the preformation factor for different fragments should also be added to this extended Hauser-Feshback method (EHFM). In a statistical model all open decay channels are taken to be

equally populated. This is true as long as one is talking of γ-decay and the light particle evaporation of n, p and α-particle. However, once the heavy fragments are also included, α-particle is known to compete with some heavy fragments (the exotic cluster decays) and hence, a preformation probability factor between α- and the other heavy-fragments should come in.

In this paper, we use the quantum mechanical fragmentation theory, the QMFT [13-16], and the cluster decay model [1,21,22] based on QMFT. According to the QMFT, the binary fragmentation is a collective mass transfer process where both the light and heavy fragments (including the light particles) are produced with different quantum mechanical probabilities. Applications of both the fragmenation and cluster decay processes are made in only a few cases for the light systems [13-16] and infact it was on the basis of this theory that fusion-fission was first proposed by Gupta and collaborators in 1984 as the possible explanation of the observed data on fragmentation of light systems.

In the QMFT, a dynamical collective coordinate of mass (and charge) asymmetry $\eta = \frac{A_1 - A_2}{A_1 + A_2}$ (and $\eta_Z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$) is introduced whose limiting values are 0 and 1 i.e. $0 \leq \eta \leq 1$ [14,23-27]. Since the potentials $V(R, \eta)$ and $V(R, \eta_Z)$, calculated within the Strutinsky renormalization procedure ($V = V_{LDM} + \delta U$) by using the appropriate liquid drop model (for V_{LDM}) and the asymmetric two-centre shell model (for δU), are nearly independent of the relative separation coordinate R, R can be taken as a time-independent parameter and hence solve the stationary (instead of time-dependent) Schrödinger equation in η:

$$\{-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta}\frac{1}{\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta} + V_R(\eta)\}\psi_R^{(\nu)}(\eta) = E_R^{(\nu)}\psi_R^{(\nu)}(\eta). \tag{1}$$

The R-value for light nuclei is fixed at the touching configuration of two nuclei [13-16]:

$$R = R_1 + R_2 \tag{2a}$$

with

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}. \tag{2b}$$

In this approximation, the fragmentation potential

$$V(\eta) = -B_1(A_1, Z_1) - B_2(A_2, Z_2) + E_c + V_P, \tag{3}$$

where $B_i(A_i, Z_i)$ are the experimental binding energies [28], $E_c = Z_1 Z_2 e^2 / R$ and V_P is the additional attraction due to nuclear proximity potential, given by the well known pocket formula of Blocki et. al [29]. The charges Z_i in (3) are fixed by minimizing $V(\eta_Z)$, defined by (3) without V_P, in η_Z-coordinate. The rotational energy due to angular momentum (V_l) is not added here since its contribution to the structure of yields is shown to be small for lighter systems [15]. Of course, V_l should be added for a comparison of the relative yields. It may be mentioned here that in a more realistic calculation, the two-centre nuclear shape should be used, instead of eq. (2). One can then trace the actual nuclear shapes involved. For a more quantitative comparison, perhaps $R = R_1 + R_2 + d(\le 2\text{fm})$ would be a better choice because then one is closer to the saddle shape.

The numerical solution of (1), on proper scaling, gives the fractional mass distribution yields for each fragment as

$$Y(A_i) = |\psi_R(\eta(A_i))|^2 \sqrt{B_{\eta\eta}} \frac{2}{A}, \tag{4}$$

(i= 1 or 2). For the mass parameters $B_{\eta\eta}$ we use the classical hydrodynamical model of Kröger and Scheid [30]. For two touching spheres, this model gives a simple analytical expression,

$$B_{\eta\eta} = \frac{Am R_t^2}{4} \left(\frac{v_t(1+\beta)}{v_c} - 1 \right) \tag{5a}$$

with

$$\beta = \frac{R_c}{4R_t} \left(2 - \frac{R_c}{R_1} - \frac{R_c}{R_2} \right) \tag{5b}$$

$$v_c = \pi R_c^2 R_t, \qquad R_c = 0.4 R_2 \tag{5c}$$

and $v_t = v_1 + v_2$, the total conserved volume. Also, $R_2 << R_1$ and $R_c(\ne 0)$ is the radius of a cylinder of length R_t, whose existence allows a homogenous, radial flow of mass between the two fragments. Here m is the nucleon mass.

The nuclear temperature effects in (4) are also included through a Boltzmann-like function

$$|\psi_R|^2 = \sum_{\nu=0}^{\infty} |\psi_R^{(\nu)}|^2 \, exp(-E_R^{(\nu)}/\theta) \tag{6}$$

with θ, the nuclear temperature in MeV, related to the excitation energy as

$$E^* = \frac{1}{9} A\theta^2 - \theta. \tag{7}$$

Furthermore, in some of the calculations here, temperature effects are taken to act also on the shell effects as follows [31]

$$V = V_{LDM} + \delta U exp(-\theta^2/2.25). \tag{8}$$

Similarly, mass parameters should also vary with temperature but no useable prescription is available todate. A constant average mass is taken to mean a complete washing of shell effects in it.

For cluster-decay calculations, we use the preformed cluster model (PCM) of Malik and Gupta [21,22]. This model, based on the QMFT, also uses the decoupled approximation to R- and η-motions and define the decay half-life $T_{\frac{1}{2}}$ or the decay constant λ as

$$\lambda = \frac{ln2}{T_{\frac{1}{2}}} = P_0 \nu P. \tag{9}$$

Here, P_0 is the cluster preformation probability at a fixed R, given by the solution of the stationary Schrödinger equation (1). At $R = R_1 + R_2$, $P_0 = Y(A_2)$, given by eqs. (4) and (6). For ground-state to ground-state decay, $\nu = 0$. In the following, we choose $R = R_1 + R_2$ (instead of $R = R_0$, the compound nucleus radius, where $V(R_0) = Q$-value) since this assimilates the effects of both deformations of the two fragments and neck formation between them [32].

P is the tunnelling probability, which can be obtained by solving the corresponding stationary Schrö- dinger equation in R. Instead, Malik and Gupta calculated it as the WKB penetrability which for the tunneling path shown in figure 1 of [22] is given by

$$P = P_i P_b \tag{10a}$$

with

$$P_i = exp(-\frac{2}{\hbar} \int_{R_1+R_2}^{R_i} \{2\mu[V(R) - V(R_i)]\}^{1/2} dR \tag{10b}$$

$$P_b = exp(-\frac{2}{\hbar} \int_{R_i}^{R_b} \{2\mu[V(R) - Q]\}^{1/2} dR. \tag{10c}$$

This means that tunneling begins at $R = R_1 + R_2$ and terminates at $R = R_b$ with $V(R_b) = Q$. The de-excitation probability between P_i and P_b is taken to be unity here. Both (10b) and

(10c) are solved analytically [21,22]. Apparently, we are considering here the decay from the ground-state of the parent nucleus to the ground-states of the decay products. On the other hand, if the compound system is excited or the system ends in the excited state of one or both the decay products, the Q-value has to be adjusted accordingly (as discussed in the following).

In eq. (9), ν is the assault frequency, given simply as

$$\nu = \frac{velocity}{R_0} = (2E_2/\mu)^{1/2}/R_0, \tag{11}$$

where $E_2 = (A_1/A)Q$ is the kinetic energy, taken as the Q-value shared between two fragments, and $\mu = m(\frac{A_1 A_2}{A})$ is the reduced mass.

Figure 1 shows the fragmentation potential for ^{76}Sr, plotted as a function of the light fragment mass A_2. Calculations are made in steps of one nucleon transfer. We notice in Fig. 1 that potential energy minima lie only at N=Z, A=4n nuclei, showing the strong shell effects of α-nuclei. It is important to realize here that this fragmentation potential is independent of the nuclei in the entrance channel and the light (A_2) and heavy ($A_1 = A - A_2$) fragments occur in coincidence.

The calculated fractional yields are shown in Fig. 2 for the fission of ^{76}Sr from ground-state ($\nu = 0$) and at an arbitrary $\sim$ 32 MeV of excitation energy ($\theta = 2$ MeV). We notice that in the ground-state, almost all the yield is taken away by the α-particle alone, but as the compound system is heated up, other fragments also show up. The interesting results are (i) the yields are largest for α-nuclei, and (ii) the mass distribution is strongly asymmetric. The fact that both the above mentioned results also hold good in the very small yield zone of symmetric fragmentation, we have shown in Fig. 3, the renormalized fractional yields, calculated for the fragment masses $15 \leq A_i \leq 61$ only. Apparently, the α-nuclei are favoured and the overall mass distribution is asymmetric. Also, the role of temperature in increasing the yields, more so for the symmetric and nearly symmetric fragments, is also shown in Fig. 3.

We have also analyzed the role of temperature on shell effects δU. For this purpose, we have

redone our calculations by using the theoretical binding energies [33] where V_{LDM} and δU contributions are tabulated separately. The calculated fragmentation potentials at different temperatures and the resulting yields are shown, respectively, in Figs. 4 and 5. We notice that temperature effects are large but the general character of the mass distribution remains unaffected. Our earlier calculations [15,16] show that the contribution of shell effects in mass parameters $B_{\eta\eta}$ are also of similar orders and act in the same way i.e. without disturbing the general character of the mass distribution. Following [16], the small entrance channel effects in the observed mass distributions could be assimilated by the empirically fitted $B_{\eta\eta}(\eta)$.

Our calculated decay half-lives for the decay of ^{76}Sr from its ground-state to the ground-states of all α-nuclei clusters with positive Q-value are given in Table 1. Of course, the decay could also occur into the excited states of the daughter and/or cluster nuclei. This would mean decreasing Q-value to $Q - E_i$, where, say, E_i is the energy of first excited state of daughter nucleus. This type of decay is studied in the following paragraph (Table 2). We notice in Table 1 that the penetrabilities P are very small. This is particularly so for ^{12}C decay since its Q-value is relatively very small. Apparently, all the predicted decay half-lives are very large $(T_{1/2} > 10^{80}s)$ and ^{76}Sr nucleus can be said to be stable against all possible cluster decays. Another interesting point to note is that the daughter nuclei for ^{20}Ne- and ^{32}S-decays are the doubly magic N=Z nuclei. Hence, in view of the so-far observed cluster decays [34] , these two decays should be the most probable decays. However, the preformation factors P_0 for both ^{20}Ne and ^{32}S clusters are also very small, though the decrease of P_0 with the increase of cluster size is very much in agreement with the situation for observed decays (see e.g. Fig. 13 in the review [34]).

The role of excitation energy E^* of the compound system is presented in Table 2. Once again the decays could end into the ground-states or excited states E_i of the fragments. For decay into ground-state(s) of the fragment(s), the effective Q-value (Q_{eff}) will become $Q + E^*$ but if it goes into an excited state E_1 of one fragment, $Q_{eff} = Q + E^* - E_1$. Both the cases are studied in Table 2 for some arbitrary values of E^* and the first excited states of the daughter fragments. Interesting enough, the decay constant λ increases (or $T_{1/2}$ decreases)

considerably. Such calculations for light nuclei are made for the first time and have become relevant because it is now possible to study experimentally [35] the above mentioned fine structure effects of decay into the excited states of fission fragments. Once the data becomes available, it will also help deciding between the fission and cluster decays of these light nuclear systems.

Summarizing, we have presented our calculations for fission and cluster decays of ^{76}Sr. Both the cases of ^{76}Sr in the ground-state and produced as an excited compound system in heavy-ion reactions are studied. Here fission is treated as a collective mass transfer process and cluster decay studies are based on a model allowing preformation of clusters. Calculations show that, in a clear asymmetric mass distribution, ^{76}Sr nucleus allows preferential α-nuclei transfer resonances as well as decays. The only experimental study available on this system is 1α and 2α transfer products from ^{36}Ar on ^{40}Ca at an energy near the Coulomb barrier ($V_c = 53.6$ MeV) [36]. Our calculations suggest that for obtaining the complete fission products, one has perhaps to go to at least double the Coulomb barrier energies. Also, following Sobotka et. al [11], use of inverse kinematics (projectile heavier than the target) may be an additional help. This provides a large center-of-mass velocity which facilitates the verification of full momentum transfer and easy identification of the fragment's atomic number at higher incident energies. Also, the high-enegy solution at forward angles should enhance the observation of compound-nucleus decay and virtually eliminate any possible deep-inelastic contribution.

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Figure Captions:

Fig. 1. The mass fragmentation potential for ^{76}Sr, calculated by using the experimental binding energies. Only the light fragments A_2 are shown along the x-axis. The other fragment $A_1 = A - A_2$.

Fig. 2. The calculated mass distribution yields for the fission of ^{76}Sr from the ground-state and at a temperature $\theta = 2$ MeV ($E^* \approx 32$ MeV). The range of fragment masses is $1 \leq A_i \leq 75$.

Fig. 3. The same as for Fig. 2 but at $\theta = 2$ and 3 MeV and for the range $15 \leq A_i \leq 61$. The fragmentation potential used is the same as in Fig. 1 but for the range $15 \leq A_i \leq 61$ only.

Fig. 4. The same as for Fig. 1 but by using the theoretical binding energies and for the mass range $15 \leq A_i \leq 61$ only. Also, the temperature effects on shell corrections δU are shown for $\theta = 2$ and 3 MeV.

Fig. 5. The same as for Fig. 3 but by using the potentials of fig. 4.